Übungen zu **Numerik (für Geowissenschaftler)**
Blatt 5

Ü1 (*Bases*) Let $H_{00}, H_{01}, H_{10}, and\ H_{11}$ be the polynomials of degree three, that fulfill

t	0	1		0	1		0	1		0	1
$H_{00}(t)$	1	0	$H_{01}(t)$	0	1	$H_{10}(t)$	0	0	$H_{11}(t)$	0	0
$H'_{00}(t)$	0	0	$H'_{01}(t)$	0	0	$H'_{10}(t)$	1	0	$H'_{11}(t)$	0	1

Show, that each polynomial of degree three, $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$, $a_0, a_1, a_2, a_3 \in \boldsymbol{R}$, can be written as a combination $p(t) = b_{00}H_{00}(t) + b_{01}H_{01}(t) + b_{10}H_{10}(t) + b_{11}H_{11}(t)$ with coefficients $b_{00}, b_{01}, b_{10}, b_{11} \in \boldsymbol{R}$.

Ü2 (*Newton*) Searching for the zeros of

$G : \boldsymbol{R}^2 \to \boldsymbol{R}^2$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} g_1\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) \\ g_2\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) \end{pmatrix} = \begin{pmatrix} x_2 - x_1^2 \\ x_2^2 - x_1 \end{pmatrix}$ You can use – similar to one dimensional problems – Newton's method $x^{(k+1)} = x^{(k)} - [DG(x^{(k)})]^{-1}G(x^{(k)})$

(The matrix of the derivative is $DG(x^{(k)}) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1}(x^{(k)}) & \frac{\partial g_1}{\partial x_2}(x^{(k)}) \\ \frac{\partial g_2}{\partial x_1}(x^{(k)}) & \frac{\partial g_2}{\partial x_2}(x^{(k)}) \end{bmatrix}$.)

Use different starting points!

Ü3 (*matlab*) Revisiting Ü4, sheet 1: Instead of using $\hat{x}$=`mldivide(H,b)` try to solve the linear equations with the `QR-method`, i.e. calculate $[c, R] = qr(H, b)$ and $\hat{x} = mldivide(R, c)$ (meaning: $H\hat{x} = b, H = QR, QR\hat{x} = b, R\hat{x} = Q^T b$)

Ü4 (*Iteration*) Let $A = \begin{bmatrix} 8 & 1 & 0 \\ 1 & 9 & 2 \\ -1 & 1 & 7 \end{bmatrix}$.

Calculate the norms ($|||.|||_1, |||.|||_2$ and $|||.|||_\infty$[1]) of the iteration matrices of

a) Jacobi-method

b) Gauß-Seidel-method

applied to matrix A.
For some b and $x^{(0)}$, the calculated distance yields $||x^1 - x^{(0)}|| = d$.

a) Estimate the distance $||x^\star - x^{(2)}||$.

b) How many iterates k will be sufficient, to garantee $||x^\star - x^{(k)}|| < \epsilon$
(For ex. $epsilon = 1.0e-4, 1.0e-8, 1.0e-12, .., \ d = 0.4$)

[1]Eventually with help of `matlab`