$$A_1 = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 4 & 1-\lambda & 2 \\ 1 & -2 & 2-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 1-\lambda & 2 \\ -2 & 2-\lambda \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= (1-\lambda)\Big((1-\lambda)(2-\lambda)+4\Big) - 2\,(8-4\lambda-2)$$

$$= (1-2\lambda+\lambda^2)(2-\lambda) + \underline{4} - \underline{\underline{4\lambda}} - 16 + \underline{\underline{8\lambda}} + \underline{4}$$

$$= (1-2\lambda+\lambda^2)(2-\lambda) + \underbrace{4\lambda - 8}_{4(\lambda-2)}$$

$$= (1-2\lambda+\lambda^2-4)(2-\lambda) \hookrightarrow \lambda_1 = 2$$

$$\Rightarrow \lambda^2 - 2\lambda - 3$$

$$1 \pm \sqrt{1+3} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases} \qquad \sigma = \{-1, 2, 3\}$$

EV: $\lambda = -1$

$$A + I = \begin{bmatrix} 2 & 2 & 0 \\ 4 & 2 & 2 \\ 1 & -2 & 3 \end{bmatrix} \quad v_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$\lambda = 2$

$$A - 2I = \begin{bmatrix} -1 & 2 & 0 \\ 4 & -1 & 2 \\ 1 & -2 & 0 \end{bmatrix} \quad v_2 = \begin{pmatrix} 2 \\ 1 \\ -7/2 \end{pmatrix}$$

NR: $2 \cdot 4 - 1 + 2x = 0$

Probe ✓

$\lambda = 3$

$$A - 3I \quad \begin{bmatrix} -2 & 2 & 0 \\ 4 & -2 & 2 \\ 1 & -2 & -1 \end{bmatrix} \quad v_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\sigma = \{2,\ 1+2i,\ 1-2i\}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 & 0 \\ 0 & 1-\lambda & 2 \\ 1 & -2 & 2-\lambda \end{bmatrix}$$

$\lambda = 1+2i$

$$A - (1+2i)I = \begin{bmatrix} -2i & 2 & 0 \\ 0 & -2i & 2 \\ 1 & -2 & 1-2i \end{bmatrix}$$

$$v_2 = \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix}$$

Nr: $-2i^2 + 2x$:

Probe:

$1 - 2i^2 - 1 + 2i$

ebenso

$$v_3 = \begin{pmatrix} 1 \\ -i \\ 1 \end{pmatrix}$$

$$(1-\lambda)\left(\begin{vmatrix} 1-\lambda & 2 \\ -2 & 2-\lambda \end{vmatrix}\right) + 1 \cdot \begin{vmatrix} 2 & 0 \\ 1-\lambda & 2 \end{vmatrix}$$

$$= (1-\lambda)\big((1-\lambda)(2-\lambda)+4\big) + 4$$

$$= (1-\lambda)^2 (2-\lambda) + (1-\lambda)\cdot 4 + 4$$

$$= (1-\lambda)^2 (2-\lambda) + \underbrace{\underbrace{4 - 4\lambda + 4}_{8-4\lambda}}_{4(2-\lambda)}$$

$$= \big((1-\lambda)^2 + 4\big)(2-\lambda) \hookrightarrow 2 = \lambda$$

$\lambda^2 - 2\lambda + 5$

$$1 \pm \sqrt{1-5} = 1 \pm \sqrt{-4} = 1 \pm 2i$$

$\lambda = 2$:

$$A - 2I = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 2 \\ 1 & -2 & 0 \end{bmatrix} \rightsquigarrow v_1 = \begin{pmatrix} 2 \\ 1 \\ 1/2 \end{pmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 4 & -\lambda & 2 \\ 1 & -2 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -\lambda & 2 \\ -2 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ 1 & 1-\lambda \end{vmatrix}$$

$$(1-\lambda)\Big((-\lambda)(1-\lambda)+4\Big) - 2\Big(4-4\lambda-2\Big)$$

$$(1-\lambda)^2(-\lambda) + \underline{4} \underbrace{-4\lambda \underline{-8} + 8\lambda}_{4\lambda} + \underline{4}$$

$$(-\lambda)\Big((1-\lambda)^2 - 4\Big)$$

$$\downarrow \qquad\qquad \downarrow$$

$\lambda_1 = 0$

$$\lambda^2 - 2\lambda + 1 - 4 = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda_{2/3} = 1 \pm \sqrt{1+3} = 1 \pm \sqrt{4} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$$

$$\sigma = \{0; -1, 3\}$$

EV:

$\lambda = 0:$ $\begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix}, \; v_1 = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

$\lambda = 3:$ $\begin{bmatrix} -2 & 2 & 0 \\ 4 & -3 & 2 \\ 1 & -2 & -2 \end{bmatrix}, \; v_3 = \begin{pmatrix} 1 \\ 1 \\ -1/2 \end{pmatrix}$

$\lambda = -1:$ $\begin{bmatrix} 2 & 2 & 0 \\ 4 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}, \; v_2 = \begin{pmatrix} 1 \\ -1 \\ -3/2 \end{pmatrix}$

$$A_4 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$ bzw A_α aus A1 BP WS18

$$\begin{vmatrix} \cos\alpha - \lambda & -\sin\alpha \\ \sin\alpha & \cos\alpha - \lambda \end{vmatrix}$$

$$(\cos\alpha - \lambda)^2 + (\sin\alpha)^2$$

$$= \underline{\cos^2\alpha} - 2\lambda\cos\alpha + \lambda^2 + \underline{\sin^2\alpha}$$

$$= \lambda^2 - 2\lambda\cos\alpha + 1 \; ; \quad \lambda = \cos\alpha \pm \sqrt{\cos^2\alpha - 1}$$

$$= \cos\alpha \pm \sqrt{-\sin^2\alpha}$$

$$= \cos\alpha \pm i\sin\alpha \, .$$

EV:

$$\begin{bmatrix} \cos\alpha - (\cos\alpha + i\sin\alpha) & -\sin\alpha \\ \sin\alpha & \cos\alpha - (\cos\alpha + i\sin\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} -i\sin\alpha & -\sin\alpha \\ \sin\alpha & -i\sin\alpha \end{bmatrix} \qquad v_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} i\sin\alpha & -\sin\alpha \\ \sin\alpha & i\sin\alpha \end{bmatrix} \qquad v_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$